The Color of Absorbing Scattering Fibers Having Skin-Core Structure

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SYNOPSIS

A treatment of the color of fabrics is proposed that is an extension of the Allen and Goldfinger approach to the prediction of the color of absorbing scattering substrates. Here, a model of isotropic cylindrical fibers of a skin–core structure in a parallel array is assumed. Using the proposed model, the spectral reflectance curves of ring dye and uniform dye samples are calculated. Their tristimulas values and the color difference between them are calculated to study the effect of ring distribution of dye in the fiber on color of fabric. Refractive indices of undyed and dyed polyester fibers are measured using the Pluta polarizing interference microscope. Illustration are given by microinterferograms.

INTRODUCTION

The physical conditions that enable the fibers or fabrics to absorb or to reflect in specific wavelength regions are dependent on the presence of dyes in the fibers and are determined by the physico-chemical processes of dyeing.[1]

The first thorough treatment of the color of a system simultaneously absorbing and scattering light was proposed by Kubelka and Munk in 1931.[2]

The Kubelka–Munk theory and the prediction of reflectance were studied by Nobbs.[3] Expressions were obtained for the Kubelka–Munk constants in terms of the fundamental optical parameters, and the depth of penetration of radiation into a powdered sample was investigated by Simmons.[4]

Allen and Goldfinger[5] proposed a model with which the color of fabrics and other absorbing scattering substrates can be calculated from the properties of the dye and the fiber. The properties are the geometry of the fabric, the distribution of the dye in the fiber, and the refractive indices of the continuous medium and the fibers. Cylindrical optically homogeneous fibers in a parallel array were assumed, and each "plate" consists of a parallel array of isotropic cylinders of equal diameters. Allen and Goldfinger[6] expanded this treatment to take into account the effect of an unhomogeneity of dye distribution, most easily the condition known as ring dyeing. They assumed that the refractive indices of the dyed and undyed portions of the fiber were the same.

In this work we proposed an expansion of the Allen and Goldfinger[5] model. A parallel array of cylindrical fibers with skin–core structure under the same restriction outlined in their model is assumed. Also an application of this expansion for the effect of ring dyeing on the color of fibers is carried out taking into account that the dyed and undyed portions of the fiber have different refractive indices.

MATHEMATICAL TREATMENT

Allen and Goldfinger[5] derived the following expression for the total light energy transmitted through the first pair of layers (τ_1) of their model, which had considered that the fabric was represented by a number of distinct layers of infinitely wide arrays of cylindrical fibers,

$$\tau_1 = \frac{K^2(1-a)}{(1+K)^2 - (1-a)^2} \tag{1}$$

where a is the fraction absorbed within the layer, and K is the light that had been reflected by the

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fiber surfaces or refracted within the body of the fiber. The value of K will be defined mathematically latter on.

The fraction of the incident energy reflected by this set of two arrays of fibers was given by

$$\sigma_1 = \frac{1-a}{1+K}(1+\tau_1) \tag{2}$$

In general terms they defined[5] the event within a pile of a cylindrical layers in terms of the variables

$$\tau_n = \frac{\tau_{n-1}^2}{1-\sigma_{n-1}^2} \quad \text{and} \quad \sigma_n = \sigma_{n-1}(1+\tau_n) \tag{3}$$

where the subscript n refers to the $2n$th layer of fabric.

The quantity a given in Eq. (1) was calculated and given by the following equation:

$$a = \frac{(1-\rho)(1-\alpha)}{1-\rho\alpha} \tag{4}$$

where ρ is the Fresnel refraction at the fiber surface. It has two values,[5] which have to be handled separately for the parallel and perpendicular polarization components of the radiation. α is defined from Beers–Lambert's law as follows:

$$\alpha = 10^{-ckl_p} \tag{5}$$

where ck is the product of the concentration of colorant and the coefficient of absorption per unit radius; l_p is path length of light thought the fiber between the points of internal refraction.

In the present work cylindrical fibers having skin-core structure in a parallel array are assumed (see Fig. 1). n_s and n_c are the refractive indices of fiber's skin–core, respectively.

Let θ_0 be the angle of incidence of light (see Fig. 2), θ_1 and θ_2 are the angles of refraction and incidence of light in the skin and core layers, respectively. θ_3 is the angle of refraction inside the core; e is the thickness of the skin. Same relations for the values τ_n, σ_n, and a given by Eqs. (3) and (4) can be derived in our skin–core model taking into account the limiting assumption that the reflectance at the skin–core interface may be neglected.

The Calculation of α

As can be seen in Figure 2 the value of ckl_p represented in Eq. (5) is given by

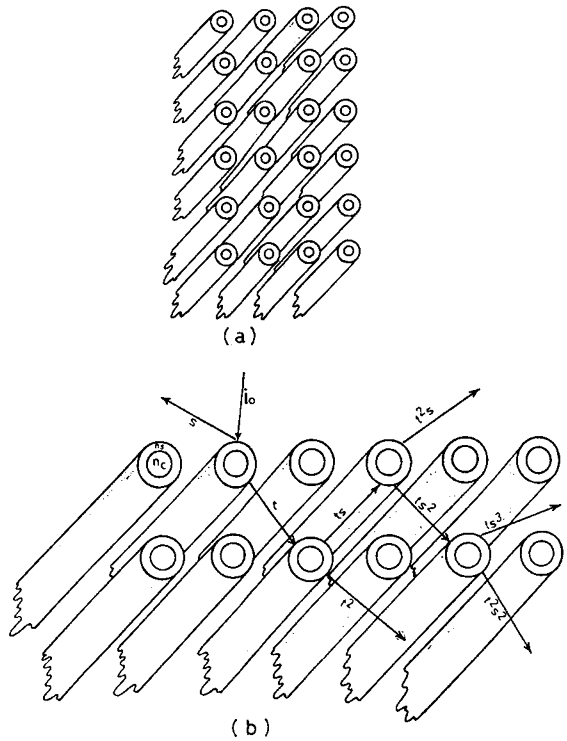

Figure 1 (a) Cross section through model of fabric have skin–core structure. (b) Representation of grouping of absorbing scattering arrays for the first iteration: t is a fraction of the incident flux on the first layer in the general direction of the incidence flux i_0; s is the fraction of incident flux having its direction reversed.

$$ckl_p = ck_s l_{ps} + ck_c l_{pc} \tag{6}$$

where l_{ps} and l_{pc} are the path lengths of light through the skin, and core, respectively; ck_s and ck_c are the products of concentration of colorant and the coefficient of absorption for skin and core, respectively. One concludes from Figure 2 the following equation for the path length of light l_{pc} in the core:

$$l_{pc} = 2(1-e)\cos\theta_2$$

Hence,

$$l_{pc} = \frac{2}{n_c}[n_c^2(1-e)^2 - d^2]^{1/2} \tag{7}$$

Also as can be seen from Figure 2 the path length of light through the skin is given by the following equation:

$$l_{ps} = \frac{2(1-e)\sin(\theta_2-\theta_1)}{\sin\theta_1}$$

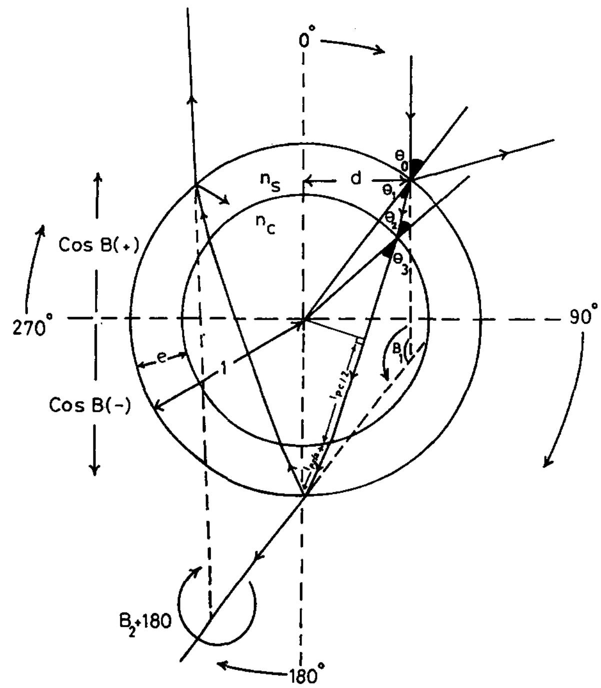

Figure 2 Cross section through a fiber having a skin–core structure, showing the different light reflections and refractions.

Hence,

$$l_{ps} = \frac{2}{n_s}\sqrt{n_s^2 - d^2} - \frac{2}{n_s}\sqrt{n_s^2(1-e)^2 - d^2} \quad (8)$$

From Eqs. (6)–(8) one can get the following equation:

$$\alpha = 10^{-[ck_s(\sqrt{(1-d^2/n_s^2)} - \sqrt{(1-e)^2 - d^2/n_s^2}) + ck_c\sqrt{(1-e)^2 - d^2/n_c^2}]} \quad (9)$$

Throughout the calculation, Eq. (4) will be handled separately for $\rho^{\parallel}$ and $\rho^{\perp}$ and numerical integration will be carried out for the values of a calculated from $d = 0$ to $d = 1$, where d is the distance from y axis.

The Calculation of K

Using the assumption that the reflectance at the skin–core interface is neglected, one can get the same expression derived by Allen and Goldfinger,[5]

$$K = \frac{\rho_{d=0.707\to 1} + (1+\rho)^2 \sum_{n=1}^{\infty} \alpha^n \rho^{n-1}_{\cos\beta_n(-)}}{\rho_{d=0\to 0.707} + (1+\rho)^2 \sum_{n=1}^{\infty} \alpha^n \rho^{n-1}_{\cos\beta_n(+)}} \quad (10)$$

In our skin–core model note that Eq. (10) has the following consederations:

1. K weighted for parallel and perpendicular polarized portions of the radiation (with the appropriate values substituted for ρ, s).
2. In the present model α is the quantity defined in Eq. (9).
3. The auxiliary quantity β is used to calculate the direction of refracted radiation in the fiber. It can be calculated as follows.

As can be seen from Figure 2, the auxiliary quantity β_1 is given by the following expression:

$$\beta_1 = 180 - 2[(\theta_0 - \theta_1) + (\theta_2 - \theta_3)]$$

Then,

$$\cos\beta_1 = -\cos 2[(\theta_0 + \theta_2) - (\theta_1 + \theta_3)]$$

Hence, one can calculate $\cos\beta_1$ and summarized its value in the following equation:

$$\begin{aligned}
\cos\beta_1 = {} & [-1/n_s^4 n_c^2 (1-e)^4] \\
& \times \{[(1-2d^2)(n_s^2(1-e)^2 - 2d^2) - 4d^2 \\
& \quad \times (1-d^2)^{1/2}(n_s^2(1-e)^2 - d^2)^{1/2}] \\
& \quad \times [(n_s^2 - 2d^2)(n_c^2(1-e)^2 - 2d^2) - (4d^2 \\
& \quad \times (n_s^2 - d^2)^{1/2}(n_c^2(1-e)^2 - d^2)^{1/2}] \\
& \quad + [2d(n_s^2(1-e)^2 - 2d^2)(1-d^2)^{1/2} \\
& \quad + 2d(1-2d^2)(n_s^2(1-e)^2 - d^2] \\
& \quad \times [2d(n_c^2(1-e)^2 - 2d^2)(1-d^2)^{1/2} \\
& \quad\quad + (2d(n_s^2 - 2d^2)(n_c^2(1-e)^2 - d^2)^{1/2}]\} \quad (11)
\end{aligned}$$

For these consequence $\beta_2 = \beta_1 + [180 - 2(\theta_1 + \theta_3)]$ subsequent refraction, $N = 1, 2, 3, \ldots$, increase the angle β_N by the same value $[180 - 2(\theta_1 + \theta_3)]$ so that

$$\beta_N = \beta_{N-1} + [180 - 2(\theta_1 + \theta_3)]$$

Then, $\cos\beta_N = \cos[\beta_{N-1} - 2(\theta_1 + \theta_3)]$. Hence, we can get the following expression:

$$\begin{aligned}
\cos\beta_N = {} & [1/(n_s^2 n_c^2(1-e)^2] \\
& \times \{\cos\beta_{N-1}[(n_s^2 - 2d^2)(n_c^2(1-e)^2 - 2d^2) \\
& + (2d(1-d^2)^{1/2}(2d(n_c^2(1-e)^2 - d^2)^{1/2}]
\end{aligned}$$

$$+ \sin \beta_{N-1}[2d(n_s^2 - d^2)^{1/2}(n_c^2(1 - e^2) - 2d^2) + (2d(n_c^2(1 - e)^2 - d^2)^{1/2}(n_s^2 - 2d^2)]\} \quad (12)$$

The Effect of the Ring Dyeing

In the case where a given amount of dye is distributed uniformly in a homogeneous isotropic cylindrical fiber, it has a refractive index n. If the same amount of dye is concentrated in a ring of thickness e, then the fiber will have two refractive indices, one refers to the ring n_s and the other to the core n_c. Allen and Goldfinger assumed in their model[5] that no changes in the refractive index of ring and core of the fiber, i.e., $n_s = n_c$, which is not valid with experimental evidence.[7] In our case the previous model of skin–core can be applied for the ring dye model taking into account the following conseder-ation:

1. The concentration c times the coefficient of absorption k of the dye for the ring is given by

$$ck_s = \frac{ck}{e(2 - e)} \quad (13)$$

and if it is found in the core, it becomes

$$ck_c = ck(1 - e)^2 \quad (14)$$

2. The path length l_p given in Eq. (5) has the following value in the case of the ring dye model:

$$l_p = l_{ps} = \frac{2}{n_s}\left\{(n_s^2 - d^2)^{1/2} - \left[(1 - e^2) - \frac{d^2}{n_s^2}\right]^{1/2}\right\}$$

RESULTS

Interferometric measurements of the refractive indices of undyed and dyed polyester fibers with Resoline Violet RL dyestuff are carried out to support the validity of our model. This experimental measurements is given to study the changes of the refractive index of polyester fibers after dyeing. Hence in the case of ring dye, the fiber will have two refractive indices, one refers to that of the ring and the other to that of the core.

The totally duplicated images of the fiber using the Pluta[8,9] polarizing interference microscope is used to measure the mean refractive indices of undyed and dyed polyester fibers with Resoline Violet RL dyestuff. Two fibers are placed on a microscope slide (the undyed polyester fiber and the dyed one) close and parallel to each other, and their ends are fixed to the slide by an adhesive. A drop of a suitable liquid is placed on the slide such that the fibers are immersed in it. A cover slip is placed on the two fibers. The objective prism of the Pluta polarizing interference microscope is rotated to obtain the maximum duplication of the two images of the fiber, and the slit diaphragm is rotated to make an angle of 135° with axis to obtain the sharpest fringes. Two parallel images of each fiber appeared in the field of view. These images are perpendicular to the interference fringes. The fringe displacement of the upper image of the fiber is caused by the difference in refractive index $(n_a^{\parallel} - n_L)$, whereas the fringe displacement in the lower image is due to the refractive index difference $(n_a^{\perp} - n_L)$, where n_L is the refractive index of the liquid and n_a is the refractive index of the fiber for light vibrating parallel or perpendicular to the fiber axis.

Figure 3 is a microinterferogram of totally duplicated images of a polyester fiber using monochromatic light of wavelength $\lambda = 546$ nm. An immersion liquid of refractive index $n_L = 1.613$ at temperature 27°C is used. The upper fiber is a dyed polyester fiber with Resoline Violete RL dyestuff. The lower is the undyed polyester fiber. Figure 4 [(a) and (b)] are microinterferograme of nonduplicated image using the Pluta microscope for undyed and dyed

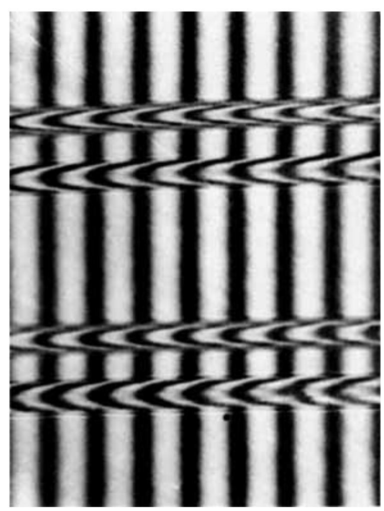

Figure 3 Totally duplicated images of undyed and dyed polyester fibers using the Pluta microscope with monochromatic light of wavelength 546 nm. The upper fiber is the undyed one and the lower fiber is the dyed ones.

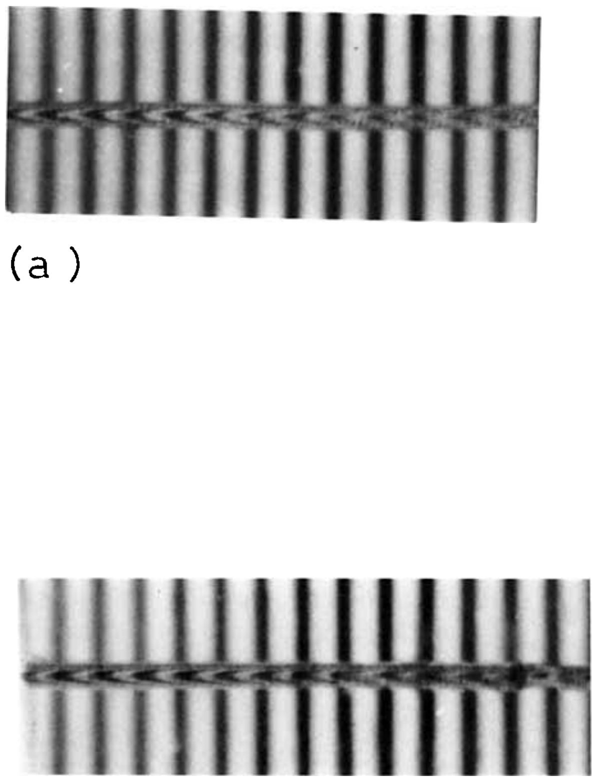

Figure 4 Microinterferograme of nonduplicated image of a polyester fiber using the Pluta microscope with monochromatic light of wavelength 546 nm. (a) Undyed polyester fiber; (b) polyester fiber dyed with Resoline Violete RL dyestuff.

polyester fibers, respectively. A monochromatic light of wavelength $\lambda = 546$ nm is used and the immersion liquid = 1.6455. The calculation of the mean refractive indices for undyed and dyed polyester fibers are carried out using the following formula (cf. Ref. 10):

$$n_a = n_L + \frac{\lambda}{Mh}\frac{z_a}{t_a}$$

where n_L is the refractive index of the immersion liquid, λ is the wavelength of light used, Z_a is the interference fringe shift inside the tested fiber, M is the magnification, h is the interfering spacing, and t_a is the thickness of this fiber. These calculations are carried out for several fibers, and the average value is taken into consederation. Table I gives the mean refractive indices $n^{\parallel}$ and $n^{\perp}$ of undyed and dyed polyester fibers for light vibrating parallel and perpendicular to the fiber axis. Also the birefringence (Δn) is given in the same table.

Table I Refractive Indices and Birefringence of Undyed and Dyed Polyester Fibers Using an Immersion Liquid of Refractive Index n_L = 1.613

Fiber	$n^{\parallel}$	$n^{\perp}$	Δn
Polyester fibers	1.6920	1.552	0.140
Polyester fibers dyed with Resoline Violete RL dyestuff	1.7100	1.552	0.158

It is clear that the dying processes changes the mean refractive index of the fiber. Hence, in the case of fiber have ring dying, its refractive indices of the ring dye and undyed core portions of the fiber are of different values.

Using Eqs. (1)–(4) and (9)–(14), the reflectance R (R is used instead of σ) of fibers having skin–core structure is calculated. The reflectance R is calculated as a function of ck. The numerical summation of the σ_n values ($\sigma_n = R$) was extended to $\tau_n < 10^{-6}$ σ_n. A limiting assumption $(1 \leq e \leq n_s - 1/n_s)$ is considered.

Figure 5 shows the relation between the reflectance R, of an array of fibers having a skin–core structure, against ck. Also the parallel ($R^{\parallel}$) and perpendicular ($R^{\perp}$) components of this reflected light flux are given in Figure 5.

An application of this model to the effect of ring dyeing on color of fibers is carried out taking into account that the inhomogeneity of dye distribution causes a skin–core structure with different refractive indices. The reflectance R_r is calculated as a function

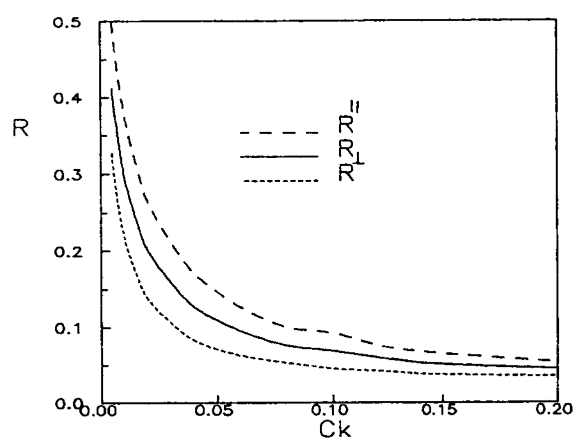

Figure 5 The reflectance R of an array of fibers having a skin–core structure plotted against ck. The refractive indices of skin and core (n_s and n_c) of fibers are 1.5 and 1.6, respectively.

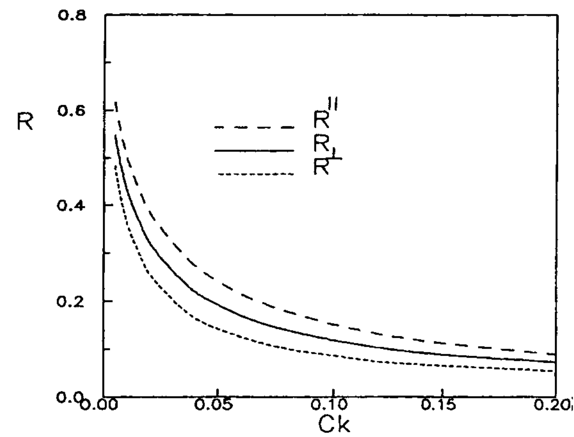

Figure 6 Reflectance as a function of ck for an array of ring-dyed fibers of ring 0.1 radius thick. Data are for $n_s = 1.5$ and $n_c = 1.6$.

of ck for an array of fibers having a ring of dye and shown in Figure 6. Also the parallel ($R^{\parallel}$) and perpendicular ($R^{\perp}$) components of this reflected light flux are shown in Figure 6.

To study the effect of a ring distribution of dye in the fiber on the color of fabric quantitatively, the spectral reflectance curves of three different colors are calculated theoretically using our ring-dyed model.

In this calculation we used the spectral transmittance values of dye–fiber polymer system with the refractive indices of the ring and core fibers and their constants of the Cauchy's dispersion formula, and a thickness of the ring dye. Also the reflectance curves of homogeneous dye distribution for the same fiber with the same dye concentration are calculated.

These calculations are carried out for red, green, and blue colors. Figure 7 [(a), (b), (c)] shows the absorption curves of the used dyestuffs.[11] The calculated spectral reflectance curves of ring dye, and homogeneous distribution dye of equal dye concentration 1% are shown also in Figure 7 [(a), (b), (c)]. We assume that the diameter of fibers is of 100 μm and its Cauchy's constants are $A_s = 1.5$ and $B_s = 21210$ nm^2 for the ring, $A_c = 1.6$ and $B_c = 21210$ nm^2 for the core, and $A = 1.6$ for uniform dye distribution samples. The obtained curves show that the ring-dyed samples reflect more light than uniform dyed samples. These theoretical calculations are supported by experimental results.[7]

To evaluate this effect quantitatively the tristimulas values X, Y, Z are calculated for the three colors of both ring dye and uniform dye reflectance curves and the color difference $\Delta E(L^*, a^*, b^*)$[12] values

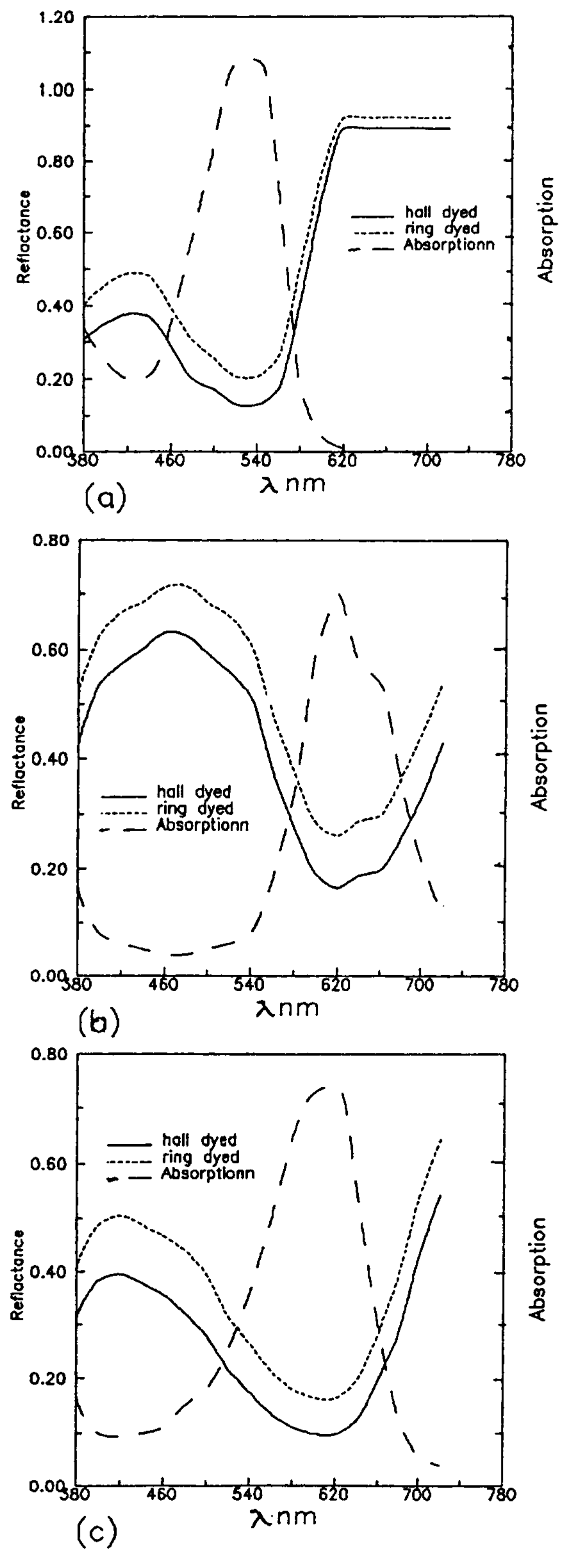

Figure 7 Spectral reflectance curves of a ring-dyed and uniform dyed fibers, and the absorption curve of the used dyestuffs: (a) red; (b) green; and (c) blue dyestuff. (The absorption curves are taken from Ref. 11.)

Table II Tristimulas Values of the Ring-dyed and the Uniform Dyed Samples, and the Color Difference Between Them for the Colors Red, Green, and Blue

Dyestuff	Model	X	Y	Z	x	y	ΔE (L*a*b*)
Red	Ring dye	56.118	40.949	48.388	0.386	0.282	—
	Uniform dye	44.871	31.001	36.073	0.401	0.277	8.235
Green	Ring dye	42.539	50.933	80.827	0.244	0.929	—
	Uniform dye	33.217	41.210	70.795	0.229	0.284	7.634
Blue	Ring dye	24.710	26.241	54.442	0.234	0.249	—
	Uniform dye	17.011	17.955	41.789	0.222	0.234	9.128

between them are calculated for different used dyestuffs. The results of X, Y, Z, and ΔE value are given in Table II. It is clear that the Y value in the case of ring-dyed sample is higher than that of the uniform dyed sample, and a significant color difference occurs between them.

CONCLUSION

A model of fibers of skin–core structure in a parallel array is assumed. A treatment of the color of a fabrics is carried out as an extension of the Allen and Goldfinger approach to the prediction of the color of absorbing scattering substratas. The measured refractive indices of undyed and dyed polyester fibers show that the mean refractive index of dyed polyester fiber is higher than of that of undyed ones. Our treatment for the colors of fibers (skin–core structure) has useful applications for the ring-dyed fibers because the inhomogeneity of dye distribution in fibers. In this treatment we omit the approximation given by Allen and Goldfinger model for ring dyeing in which they assumed that the refractive indices of the dyed and undyed portion of the fiber are of the same values. In our opinion this treatment is more satisfactorily than that given by Allen and Goldfinger.

The results show that the ring-dyed samples reflected more light than homogeneous dyed samples. Allen and Goldfinger[13] gave experimental results that supports the validity of our approach.

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